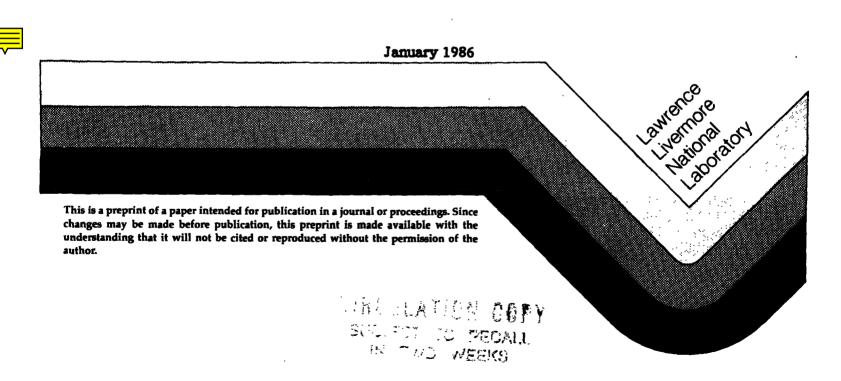
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B. A. Lippmann

This paper was prepared for submittal to Phys. Rev. A – Brief Reports



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ABSTRACT

The index of refraction of a free electron laser (FEL) pulse is derived directly, independently of the general FEL analysis, by considering the propagation of an electromagnetic wave along an electron beam that is penetrated by a static magnetic field.

^{*}Consultant, LLNL. Permanent address 215 El Verano, Palo Alto, CA 94306.

Recent work¹ on the optical guidance of a free electron laser (FEL) pulse utilized the index of refraction of the pulse, as calculated in the course of a theoretical analysis of FEL behavior.² However, if the FEL electron pulse is regarded as the propagation medium for an electromagnetic wave, the index of refraction is a basic characteristic of the medium. As such, it may be calculated directly, independently of the general FEL analysis.

To show this, consider the following one-dimensional problem: an electron beam, moving in the z-direction with a velocity close to c, is penetrated by a static magnetic field, periodic in z and oriented transversely to z; what index of refraction does this medium present to an electromagnetic wave propagating along z?

Suppose the wiggler field, B_w , is oriented along x and the propagating electric field E_s is oriented along y. Then, the fields can be derived from two vector potentials, 3 oriented along y, given by

$$A_{\mathbf{W}} = B_{\mathbf{W}} \frac{\sin \int \mathbf{k}_{\mathbf{W}} d\mathbf{z}}{\mathbf{k}_{\mathbf{W}}}$$

and

$$A_{S} = E_{S} \frac{\sin \psi_{S}}{k_{S}}$$

where,

$$\psi_{\rm S} = k_{\rm S} z - w_{\rm S} t + \varphi(z)$$

Since there is no variation transverse to z, the canonical momenta along x and y are constants of the motion, which, as in reference 2, we take to be zero.

The y-velocity induced in the electron beam can be obtained directly from the canonical momentum along y:

$$v_y = \frac{-e}{\gamma mc} (A_w + A_s)$$

and the polarization current along y, is $(\eta = index \text{ of refraction})$

$$\text{nev}_{y} = \frac{\hat{p}}{4\pi} = \frac{(\eta^{2}-1)}{y} = \frac{\hat{E}}{4\pi} = \frac{(\eta^{2}-1)}{c} \quad (-\hat{A}_{s})$$

or, writing $\omega_{\rm p}$ for the plasma frequency,

$$(\eta^2 - 1) A_s = -\frac{\omega_p^2}{\omega_s^2} (A_w + A_s)$$

which can be put in the form $(A_w >> A_s)$,

$$(\eta^2-1) \frac{E_s}{k_s} (1-e^{-2i\psi_s}) = \frac{\omega_p^2}{\omega_s^2} \frac{e^{-i\psi}}{\gamma} \frac{B_w}{k_w} (1-e^{2i\int k_w dz})$$

with

$$\psi = \int k_{W} dz + \psi_{S} \qquad .$$

Prosnitz et. al assume that ψ varies slowly, while ψ_s and $\int k_w \, dz - \psi_s$ vary rapidly. The rapidly varying terms may be averaged out, leaving

$$\eta^2 - 1 = \frac{\omega_p^2}{\omega_s^2} \frac{e^{-i\psi}}{\gamma} \frac{B_w k_s}{E_s k_w}.$$

If, as in the FEL, $\eta \sim 1$, η^2-1 can be replaced here by $2(\eta-1)$. Since η can be complex, taking real and imaginary parts reproduces the results in reference 1, namely,

$$(\eta-1) = \frac{\omega_p^2}{2\omega_s^2} < \frac{e^{-i\psi}}{\gamma} > \frac{B_w^2 k_s}{E_s^2 k_w}$$

where the brackets <...> represent an average over all electrons in the pulse.

I thank Phil Morton for entertaining conversations peppered with useful comments.

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

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- (a) N. M. Kroll, P. L. Morton, and M. W. Rosenbluth, IEEE J. Quantum Electron. <u>17</u>, 1436 (1981); (b) D. Prosnity, A. Szöke, and V. K. Neil, Phys. Rev. <u>A24</u>, 1436 (1981).
- 3. These fields are as in reference 2b, except that we use Gaussian units.

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